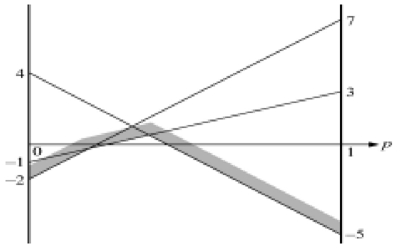
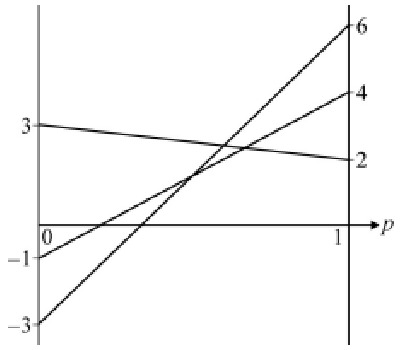
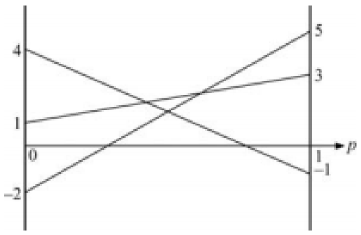
JUNE 2010

4(a)(i)	Let Roger play R_1 with probability p and R_2 with probability $1-p$			
	Expected gains:			
	$C_1 : 7p - 2(1-p) = 9p - 2$	M1		one correct unsimplified
	$C_2 : 3p - (1-p) = 4p - 1$			
	$C_3 : -5p + 4(1-p) = 4 - 9p$	A1		all correct unsimplified
		M1 A1		2 of their lines drawn correctly all correct and accurate for $0 \le p \le 1$ Condone lines not quite to p = 1 if using “accurate” intersection points on p-axis i.e. $\frac{2}{9} < \frac{1}{4}$ and $\frac{4}{9} \approx twice \frac{2}{9}$
	C_2 and C_3 lines give optimum			
	$4p - 1 = 4 - 9p$	M1		ft their max point of region
	$p = \frac{5}{13}$	A1		Condone 0.385 or 0.3846(15…) must be correct rounding if 3sf used
	Roger plays			
	R_1 $\frac{5}{13}$ of time and R_2 $\frac{8}{13}$ of time	E1	7	CAO
(ii)	Value of game $= 4 \times \frac{5}{13} - 1 = \frac{7}{13}$	B1	1	AG or $\left(4 - 9 \times \frac{5}{13}\right) = \frac{7}{13}$ must see correct calculation
(b)	Let Corrie play C_1 with prob p, C_2 with prob q, C_3 with prob $1-p-q$			
	$R_1 : 7p + 3q - 5(1-p-q)$	M1		any correct expression
	$R_2 : -2p - q + 4(1-p-q)$			
	$\Rightarrow 12p + 8q = 5\frac{7}{13}$	A1		either equation correctly with coefficients of p and q correctly simplified
	$6p + 5q = 3\frac{6}{13}$			
	$\Rightarrow \left.\begin{matrix} q = \frac{9}{13} \\ p = 0 \end{matrix}\right\}$	m1 A1CSO		*may reason that $p(C_1) = 0$ from part(a)E1 with M1, A1, A1, E1 from 2 × 2 equations* $3r - 5s = \frac{7}{13}$ $-r + 4s = \frac{7}{13}$
	$\Rightarrow$ Optimal mixed strategy is			
	C_1 with prob 0			
	C_2 with prob $\frac{9}{13}$			Condone 0.692
	C_3 with prob $\frac{4}{13}$	E1	5	CAO & 0.308
	Total		**13**	

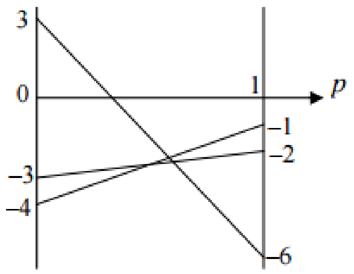
JAN 2011

3(a)(i)	Row minima 2, –3, x	B1	1	Check for answers written on table
(ii)	Column maxima 3, 6, 4	B1		
	Max (row min) = 2 Min (col max) = 3 Or $2 \neq 3$	M1		Condone Best (worst) =2 etc Worst (best) =3
	Since $2 \neq 3 \rightarrow$ no stable solution	A1cso	3	Both lines and statement must score previous B1, B1
(b)	$x < 2, x+3 < 6, 3 < 4$ $\rightarrow R_1$ dominates R_3 } Either of these	B1	1	hence Rhona should not play R_3
(c)(i)	Let Rhona play R_1 with prob p and R_2 with prob $1-p$			
	When C plays C_1: exp value $= 2p + 3(1-p)$ C_2: $6p - 3(1-p)$ C_3: $4p - (1-p) = -1 + 5p$	M1 A1		$= 3 - p$ $= -3 + 9p$ any two correct unsimplified all correct unsimplified
		M1 A1		drawing two of their expected values for $0 \leq p \leq 1$ both vertical axes using same scale condone use of horizontal lines in paper all three correct lines must see numbers on at least one vertical axis
	$3 - p = -1 + 5p$ $\rightarrow p = \frac{2}{3}$	M1 A1		choosing highest point of region
	$\rightarrow$ Rhona plays R_1 $\frac{2}{3}$ of time and R_2 $\frac{1}{3}$ of time	E1√	7	ft their p
(ii)	Value of game $= 3 - \frac{2}{3} = \frac{7}{3}$	B1	1	or $-1 + \frac{10}{3} = \frac{7}{3}$
	Total		**13**	

JUNE 2011

3(b)(i)	Let Rohan play R_1 with prob p $\Rightarrow$ plays R_2 with prob $1-p$ When Carla plays C_1, Rohan's expected gain $=3p+(1-p)$ $=1+2p$			
	$C_2: 5p+(-2)(1-p)=7p-2$	M1		at least 2 expected gains correct unsimplified
	$C_3: -p+4(1-p)=4-5p$	A1		all 3 correct unsimplified
		M1		at least 2 lines correct
		A1		all lines correct for $0 \leqslant p \leqslant 1$ and values at 0 and 1 clear
	$7p-2=4-5p$ $12p=6$	M1		choosing highest point or using correct equation
	$\Rightarrow p=\frac{1}{2} \Rightarrow$ Rohan plays R_1 50% of the time and R_2 50% of the time	A1cso		
	Value of game $=7\times\frac{1}{2}-2=\frac{3}{2}$ AG	B1	7	or $4-\frac{5}{2}=\frac{3}{2}$ must see working
(b)(ii)	When Rohan plays R_1, expected loss for Carla is $3p+5q+(-1)(1-p-q)$ and when Rohan plays R_2, expected loss for Carla is $p+(-2)q+4(1-p-q)$	M1		either expression correct unsimplified
	$4p+6q=\frac{3}{2}+1$ $3p+6q=4-\frac{3}{2}$	A1		correct simultaneous equations unsimplified
	$\Rightarrow p=0,\ q=\frac{5}{12}$	A1		condone 0.42 or better
	$\Rightarrow$ Carla never plays C_1, plays C_2 with prob $\frac{5}{12}$ and plays C_3 with prob $\frac{7}{12}$	E1cso	4	Must have all 3 correct probabilities
	Total		**15**	

JAN 2012

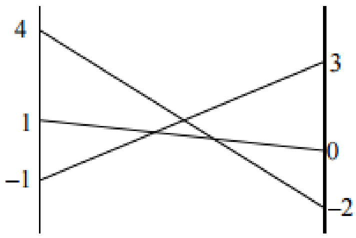
3(a)	**For each pair of strategies** Roz gain + Colum gain = 0	E2,1	2	E1 for general idea of Roz gain + Colum gain = 0
(b)	Colum's max are $-2, 3, -1$			must see these values for E1
	min (colum max) = -2	E1		
	$\Rightarrow$ play safe is C_1	B1	2	
				C_1 C_2 C_3
(c)(i)	Delete R_2 (PI by further work)	M1		-2 -6 -1 R_1
	Since R_3 dominates R_2	A1	2	-3 3 -4 R_3
(ii)	Let Roz play R_1 with prob p			
	C_1 expected gain: $-2p-3(1-p) = p-3$			
	$C_2: -6p+3(1-p) = 3-9p$	M1		2 expressions unsimplified ft their matrix
	$C_3: -p-4(1-p) = 3p-4$	A1		all correct
		M1		plotting 3 expected gains for $0 \leqslant p \leqslant 1$
		A1		correct gains plotted accurately
	Solving $p-3 = 3-9p$	m1		choosing highest point of 'their' region or correct pair solved
	$\Rightarrow 10p = 6$			
	$p = \frac{3}{5}$	A1		
	$\Rightarrow$ Roz plays R_1 with probability $\frac{3}{5}$ and R_3 with probability $\frac{2}{5}$	E1cao	7	must see R_1 and R_3
	Total		**13**	

JUNE 2012

4(a)(i)	Row min $-6, \; -3, \; -5, \; -4$ Max (row min) $= -3$	M1		attempt to find maximin and minimax condone one slip in values
	Col max $5, \; 4, \; -3$ Min (col max) $= -3$	A1		**all** rows min and col max **values correct** **and** max (row min) $= -3$ **identified** **and** min (col max) $= -3$ **identified**
	max (row min) $=$ min (col max) $= -3$ $\Rightarrow$ game has a stable solution	E1	3	**full** statement involving maximin and minimax **and** both values $= -3$
(ii)	Adam plays $\mathbf{A_2}$ & Bill plays $\mathbf{B_3}$	B1	1	
(iii)	Value of game for Bill is $+3$	B1	1	***Examiners must use the correct symbol for marks carried forward at the bottom of page 9 and top of page 10, ie ringed totals with arrows through them.***
(b)(i)	(Never play) $\mathbf{C_2}$ C_2 dominated **by** C_1 $(-3 > -4$ and $2 > 1)$	B1	1	correct strategy stated **and** correct reason condone $3 < 4$ and $-2 < -1$
(ii)	$C_1: \; 3p - 2(1-p)$ $C_3: \; -3p + 5(1-p)$	M1 A1	2	either correct unsimplified both correct unsimplified $\{5p - 2, \; 5 - 8p\}$
(iii)	$3p - 2(1-p) = -3p + 5(1-p)$ $\Rightarrow \quad p = \frac{7}{13}$	M1 A1	2	equating their 2 expressions
(iv)	Value of game $= 5 \times \frac{7}{13} - 2$ $= \frac{9}{13}$	B1	1	or $5 - 8 \times \frac{7}{13}$
	Total		**11**	

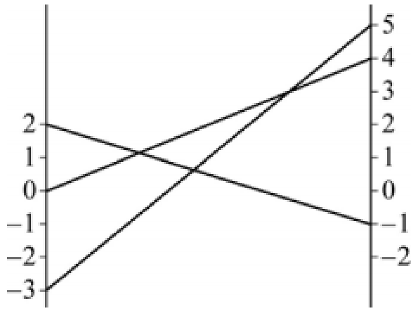
JAN 2013

2(a)	Min $\begin{pmatrix} 4 & -1 & 2 & 3 \\ 4 & 6 & 3 & 7 \\ 1 & 3 & -2 & 4 \end{pmatrix} \begin{matrix} -1 \\ 3 \\ -2 \end{matrix}$ Max 4 6 3 7			
	Maximin (row) = 3	M1		Either correct, including correct values
	Minimax (col) = 3	A1 CSO		Both correct, written as equations PI by next line
	As Maximin (row) = Minimax (col) There is a stable solution	E1		Must have equation and statement and scored first 2 marks
	(Play safe) (H) B (Play safe) (W) F	B1	4	Both correct
(b)	Saddle point (B, F)	B1	1	
	Total		**5**	

6(a)	$R_C > R_B$	E1	1	oe
(b)	$\begin{matrix} A \\ C \end{matrix}\begin{pmatrix} -2 & 0 & 3 \\ 4 & 1 & -1 \end{pmatrix}$ K plays A prob p C prob $1-p$			
	P plays D, K wins $-2p+4(1-p)$ $(=4-6p)$ E, K wins $1-p$ F, K wins $3p-1(1-p)$ $(=-1+4p)$	M1 A1		Allow 2 expressions in unsimplified form All 3 correct
		M1 A1		Must have 3 lines With values shown
	Max at $1-p=-1+4p$	M1		Identifying correct maximum from their graph
	$p=\frac{2}{5}$ (K plays) A prob $\frac{2}{5}$, C prob $\frac{3}{5}$	A1		Both stated, coming from equating correct two equations and M2 scored
	Value of game $=\frac{3}{5}$	B1	7	

6(c)	P plays D prob p E " q F " $1-p-q$			
	K plays A, P loses $-2p+3(1-p-q)=3-5p-3q$	M1		Either (unsimplified) expression correct
	K plays C, P loses $4p+q-1(1-p-q)=-1+5p+2q$			
	$3-5p-3q=\frac{3}{5}$ $-1+5p+2q=\frac{3}{5}$ $2 \quad -q=\frac{6}{5}$	m1		Equating BOTH of their expressions to value of their game
	$q=\frac{4}{5}$	A1 CSO		Or for finding p
	$5p+\frac{8}{5}-1=\frac{3}{5}$ $p=0$			
	P plays D prob 0 E, prob $\frac{4}{5}$ F, prob $\frac{1}{5}$	E1	4	All three needed, must have scored previous A mark
	Alternative method			
	Probability of D is 0	(E1)		OE, might be earned in final line
	$3(1-p)=\frac{3}{5}$ or $p-1(1-p)=\frac{3}{5}$	(M1)		Or equating the expressions
	$p=\frac{4}{5}$	(m1)		
	E prob $\frac{4}{5}$ F prob $\frac{1}{5}$	(A1) CSO		
	Total		**12**	

JUNE 2013

5(a)	R min $-4,\ -5,\ -2$ plays C J max $4,\ 1,\ 3$ plays E	B1 B1 E1	 3	Either *C* or *E* stated Both *C* and *E* stated and all values shown
(b)	maximin R $= -2 \neq 1 =$ minimax J	E1	1	Correct values must be stated
(c)	(For Juliet,) col E dominates col D	E1	1	
(d)(i)	Signs changed as J gains = R losses Gains written as rows	E1 E1	 2	
(ii)	Let J play E prob p F $(1-p)$			
	If R plays A, J wins $4p$ B $5p-3(1-p)$ C $-p+2(1-p)$ [gives $4p,\ 8p-3,\ 2-3p$]	 M1 A1		 2 correct expressions seen All correct
		m1 A1		Must have 3 lines All correct with values shown
	Max at $8p-3=2-3p$ $p=\frac{5}{11}$	m1 A1		Identifies correct max from their graph
	(J plays)E prob $\frac{5}{11}$, F prob $\frac{6}{11}$	A1 CSO	7	
(iii)	Value of game $=\frac{7}{11}$	B1	1	
	Total		**15**	

JUNE 2014

2(a)	Row min –4, 0, –5 Max (row min) = 0 Col max 5, 3, 0, 1 Min (col max) = 0	M1 A1		Attempt to find maximin and minimax Accept '*F* dominates *G*', col max 5, 3, 0 All rowmin and colmax values correct and maximin and minimax identified
	Max (row min) = Min (col max) = 0 Hence game has a stable solution.	E1		Full statement involving maximin and minimax and both values = 0 If using dominance: Reduction to 2x2 M1 Reduction to 1x1 A1 Final statement E1
	Alex plays *B* Roberto plays *F*	B1	4	
(b)	Saddle point (*B*, *F*) ONLY	B1	1	
	Total		**5**	

5(a)	*A* dominates *B*	E1	1	
(b)	Reduced matrix			

	p	q	$1{-}p{-}q$
A	4	1	–1
C	–2	0	3

E1 — Use of '$1{-}p{-}q$'

	Mark plays *A*, Owen loses $4p + q + -1(1-p-q)$	M1		One correct expression or reverse
	Mark plays *C*, Owen loses $-2p + 3(1-p-q)$	A1		Both correct or reverse
	$5p + 2q = 1.6$	m1		Correct use of 0.6 (or −0.6) Condone simplified equations
	$-5p - 3q = -2.4$	A1		2 correct equations
	$q = 0.8$ $p = 0$ $1-p-q = 0.2$	A1		At least 2 correct
	Owen plays *D* with prob 0 Owen plays *E* with prob 0.8 Owen plays *F* with prob 0.2	B1	7	All correct in context of *D*, *E*, *F*
	Total		**8**	